

## DP Curriculum Overview 2022-23

**Subject:** Mathematics

**Level & Year** DP 1 & 2: HL

*Course Description: This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof, for instance the study of sequences and series at both SL and HL, and proof by induction at HL. The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments*

### Course Aims/ Objectives:

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil

	Content	Assessment	Resources
<b>Unit 1. Measuring space and Modelling constant rate of change</b>	<p>SL 1.1* Operations with numbers in the scientific notation</p> <p>SL 1.2* Arithmetic sequences and series; use of the formulae for the <math>n</math>th term and the sum of the first <math>n</math> terms of the sequence; use of sigma notation for sums of arithmetic sequences; applications; analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.</p> <p>SL 1.3* Geometric sequences and series; use of the formulae for the <math>n</math>th term and the sum of the first <math>n</math> terms of the sequence; use of sigma notation for the sums of geometric sequences; applications such as spread of disease, salary increase and decrease, population growth.</p> <p>SL 1.4* Financial applications of geometric sequences and series including compound interest, annual depreciation.</p> <p>SL 1.5* Laws of exponents with integer exponents. Introduction to logarithms with base 10 and <math>e</math>; numerical evaluation of logarithms using technology.</p> <p>SL 1.6 Approximation: decimal places, significant figures; upper and lower bounds of rounded numbers; percentage errors; estimation.</p> <p>SL 1.6 Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side proof; the symbols and notation for equality and identity. (from AA for uniformity)</p> <p>SL 1.6 Approximation: decimal places, significant figures; upper and lower bounds of rounded numbers; percentage errors; estimation.</p> <p>SL 1.7 Amortization and annuities using technology.</p> <p>AHL 1.9 Laws of Logarithms.</p> <p>AHL 1.10 Simplifying expressions, both numerically and algebraically, involving rational exponents.</p> <p>AHL 1.11 The sum of infinite geometric sequences.</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>
<b>Unit 2: Modelling relationships and Functions</b>	<p>SL 2.2* Concept of a function, domain, range and graph; function notation; the concept of a function as a mathematical model; informal concept that an inverse function where inverse function as a reflection in the line <math>y = x</math>; the notation <math>f^{-1}</math></p> <p>SL 2.3* The graph of a function; its equation; creating a sketch from information given or a context, including transferring a graph from screen to paper; using technology to graph functions including their sums and differences.</p> <p>SL 2.4* Determine key features of graphs; finding the point of intersection of two curves or lines using technology.</p> <p>SL 2.5 Modelling with the following functions: -Linear models <math>f(x) = mx + c</math>. Including piecewise linear models -Quadratic models; identification of axis of symmetry, vertex, zeros and roots, intercepts on the <math>x</math>-axis and <math>y</math>-axis. -Exponential growth and decay models; equation of a horizontal asymptote. -Direct/inverse variation: the <math>y</math>-axis as a vertical asymptote when <math>n \neq 0</math>. -Cubic models -Sinusoidal models of the form <math>a \sin(bx) + d</math> At SL students will not be expected to translate between <math>\sin x</math> and <math>\cos x</math>. and will only be required to predict or find amplitude (<math>a</math>), period</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>

	<p>or equation of the principal axis □</p> <p>SL 2.6 Modelling skills: Use the modelling process described in the “mathematical modelling” section of the guide to create, fit and use the theoretical models in section SL2.5 and their graphs; develop and fit the model given a context, recognize and choose an appropriate model and possible parameters; determine a reasonable domain for a model. Find the parameters of a model: by setting up and solving equations simultaneously (using technology), by consideration of initial conditions or by substitution of points into a given function. Test and reflect upon the model: comment on the appropriateness and reasonableness of a model. Justify the choice of a particular model: based on the shape of the data, properties of the curve and/or on the context of the situation. Use the model: reading, interpreting and making predictions based on the model.AHL</p> <p>2.7 Composite functions in context. The notation <math>(f \circ g)(x) = f(g(x))</math>. Inverse function <math>f^{-1}</math>, including domain restriction.</p> <p>Finding an inverse function.</p>		
<b>Unit 3: Three Dimensional Space</b>	<p>SL 3.1* The distance between two points in three-dimensional space, and their midpoint; volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids; the size of an angle between two intersecting lines or between a line and a plane.</p> <p>SL 3.2* Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles; the sine rule, not including the ambiguous case; the cosine rule; area of a triangle</p> <p>SL 3.3* Applications of right and non-right angled trigonometry, including Pythagoras’ theorem. Contexts may include use of bearings; angles of elevation and depression; construction of labelled diagrams from written statements.</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>
<b>Unit 4: Analyzing rates of Change</b>	<p>SL 5.1* Introduction to the concept of a limit; derivative interpreted as gradient function and as rate of change.</p> <p>SL 5.2* Increasing and decreasing functions: graphical interpretation of signs of derivative</p> <p>SL 5.3* Derivative of functions of polynomial form or negative exponents</p> <p>SL 5.4* Tangents and normals at a given point, and their equations</p> <p>AHL 5.9 The derivatives of <math>\sin</math>, <math>\cos</math>, <math>\tan</math>, <math>e</math>, <math>\ln</math>, <math>x</math> rational power; the chain rule, product rule and quotient rules; related rates of change.</p> <p>SL 5.6 Values of <math>x</math> where the gradient of a curve is zero; solution of <math>f'(x) = 0</math>; local maximum and minimum points.</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>
<b>Unit 5: Modelling Periodic Phenomena</b>	<p>AHL 1.12 Complex numbers: the number <math>i</math> such that <math>i^2 = -1</math>; Cartesian form: <math>z = a + bi</math>; the terms real part, imaginary part, conjugate, modulus and argument; calculate sums, differences, products, quotients, by hand and with technology; calculating powers of complex numbers, in Cartesian form, with technology; the complex plane; using and drawing Argand diagrams; complex numbers as solutions to quadratic equations</p> <p>AHL 1.13 Modulus–argument (polar) form, <math>re^{i\theta}</math>; conversion between Cartesian, polar and exponential forms, by hand and with technology; calculate products, quotients and integer powers in polar or exponential/Euler forms; adding sinusoidal functions with the same frequencies but different phase shift angles for example, two AC voltages sources are</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>
<b>Unit 6: Modeling with Matrices</b>	<p>AHL 1.14 Definition of a matrix: the terms element, row, column and order for <math>m \times n</math> matrices; algebra of matrices: equality; addition; subtraction; multiplication by a scalar for <math>m \times n</math> matrices; multiplication of matrices; properties of matrix multiplication: associativity, distributivity and non-commutativity; identity and zero matrices; determinants and inverses of <math>n \times n</math> matrices with technology, and by hand for <math>2 \times 2</math> matrices; system of linear equations written in the form <math>Ax = b</math>; solution of the systems of equations using inverse matrix. Eigen Values and Eigen Vectors</p> <p>AHL 1.15 Eigenvalues and eigenvectors; characteristic polynomial of <math>2 \times 2</math> matrices; diagonalization of <math>2 \times 2</math> matrices restricted to the case where there are distinct real eigenvalues; applications to powers of <math>2 \times 2</math> matrices for example population movement, predator/prey models.</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>
<b>Unit 7: Dividing Space</b>	<p>SL 2.1* The different forms of the equation of a straight line; gradient; intercepts; parallel and perpendicular lines.</p> <p>SL 3.5 Equations of perpendicular bisectors. SL 3.6 Voronoi diagrams; sites, vertices, edge, cells; addition of a site to an existing Voronoi diagram; nearest neighbor interpolation; applications including the “toxic waste dump” problem.</p>	<p>Formative assessments</p> <p>Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL</p> <p>Haese and Harries AASL</p> <p>Khan Academy Videos</p> <p>InThinking</p> <p>Desmos</p> <p>Geogebra</p> <p>Excel</p> <p>TI Inspire GDC</p> <p>Waterloo Website</p>



<p><b>Unit 8: Optimising Complex networks</b></p>	<p>AHL 3.14 Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges, degree of a vertex; simple graphs; complete graphs; weighted graphs; directed graphs; indegree and outdegree of the vertices of a directed graph; subgraphs; trees          AHL 3.15 Adjacency matrices; walks; number of k-length walks (or less than k-length walks) between two vertices; weighted adjacency tables; construction of the transition matrix for strongly-connected, undirected or directed graphs.          AHL 3.16 Tree and cycle algorithms with undirected graphs; walks, trails, paths, circuits, cycles; Eulerian trails and circuits; Hamiltonian paths and cycles; minimum spanning tree (MST) graph algorithms; Kruskal's and Prim's algorithms for finding minimum spanning trees; use of matrix method for Prim's algorithm; Chinese postman problem; Travelling salesman problem; nearest neighbour algorithm for determining an upper bound for the travelling salesman problem; deleted</p>	<p>Formative assessments          Paper 1, 2 and Paper 3 style questions on unit end summative assessment</p>	<p>Oxford publication AASL          Haese and Harries AASL          Khan Academy Videos          InThinking          Desmos          Geogebra          Excel          TI Inspire GDC          Waterloo Website</p>
---	---	---	--

	Content	Assessment	Resources
<p><b>Unit 9: Representing multiple outcomes and testing validity</b></p>	<p>SL 4.1* Concepts of population, sample, random sample, discrete and continuous data; reliability of data sources and bias in sampling; interpretation of outliers (where outlier defined as a data item which is more than <math>1.5 \times</math> interquartile range (IQR) from the nearest quartile); sampling techniques: simple random, convenience, systematic, quota and stratified.          SL 4.2* Presentation of data (discrete and continuous); frequency histograms with equal class intervals; cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR); production and understanding of box and whisker diagrams; use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile range or range; determining whether data may be normally distributed by consideration of the symmetry of the box and whiskers.          SL 4.3* Measures of central tendency (mean, median and mode); estimation of mean from grouped data; modal class; measures of dispersion (interquartile range, standard deviation and variance); effect of constant changes on the original data; quartiles of discrete data.          SL 4.4* Linear correlation of bivariate data; Pearson's product-moment correlation coefficient, <math>r</math>; scatter diagrams; lines of best fit, by eye, passing through the mean point; equation of the regression line of <math>y</math> on <math>x</math>; use of the equation of the regression line for prediction purposes; interpret the meaning of the parameters, <math>a</math> and <math>b</math>, in a linear regression <math>y = ax + b</math>.          SL 4.5* Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (<math>U</math>) and event; the simple probability of an event <math>A</math>, the complementary events <math>A</math> and <math>A'</math> (not <math>A</math>); expected number of occurrences.          SL 4.6* Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities; combined events; mutually exclusive events, conditional probability; independent events; Problems can be solved with the aid of a Venn diagram, tree diagram, sample space diagram or table of outcomes without explicit use of formulae.          SL 4.7* Concept of discrete random variables and their probability distributions; expected value (mean), <math>E(X)</math> for discrete data; applications.          SL 4.8* Binomial distribution; situations where the binomial distribution is an appropriate model; mean and variance of the binomial distribution.</p>	<p>Formative assessments          Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL          Haese and Harries AASL          Khan Academy Videos          InThinking          Desmos          Geogebra          Excel          TI Inspire GDC          Waterloo Website</p>
<p><b>Unit 10: Vectors</b></p>	<p>AHL 3.10 Concept of a vector and a scalar; representation of vectors using directed line segments; unit vectors; base vectors <math>i, j, k</math>; components of a vector; column representation; the zero vector <math>0</math>, the vector <math>\vec{v}</math>; position vectors; rescaling and normalizing vectors, for example, finding the velocity of a particle with speed <math>7 \text{ ms}^{-1}</math> in the direction <math>3i + 4j</math>          AHL 3.11 Vector equation of a line in two and three dimensions <math>r = a + b</math>          AHL 3.12 Vector applications to kinematics; finding positions, intersections, describing paths,</p>	<p>Formative assessments          Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL          Haese and Harries AASL          Khan Academy Videos          InThinking          Desmos          Geogebra</p>
<p><b>Unit 11: Approximating irregular spaces</b></p>	<p>AHL 5.15 Slope fields and their diagrams.          AHL 5.16 Euler's method for finding the approximate solution to first order differential equations. Numerical solution of <math>dy/dx = f(x,y)</math>; numerical solution of the coupled system of the form <math>dx/dt = f_1(x,y,t)</math> and <math>dy/dt = f_2(x,y,t)</math>; contexts including predator-prey models          AHL 5.17 Phase portraits for the solutions of coupled differential equations of the form <math>dx/dt = ax + by</math>; qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues; sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.          AHL 5.18 Solutions of <math>d^2x/dt^2 = f(x, dx/dt, t)</math> Euler's method.</p>	<p>Formative assessments          Paper 1 and Paper 2 style questions on unit end summative assessment</p>	<p>Oxford publication AASL          Haese and Harries AASL          Khan Academy Videos          InThinking          Desmos          Geogebra          Excel          TI Inspire GDC          Waterloo Website</p>

D  
P  
2