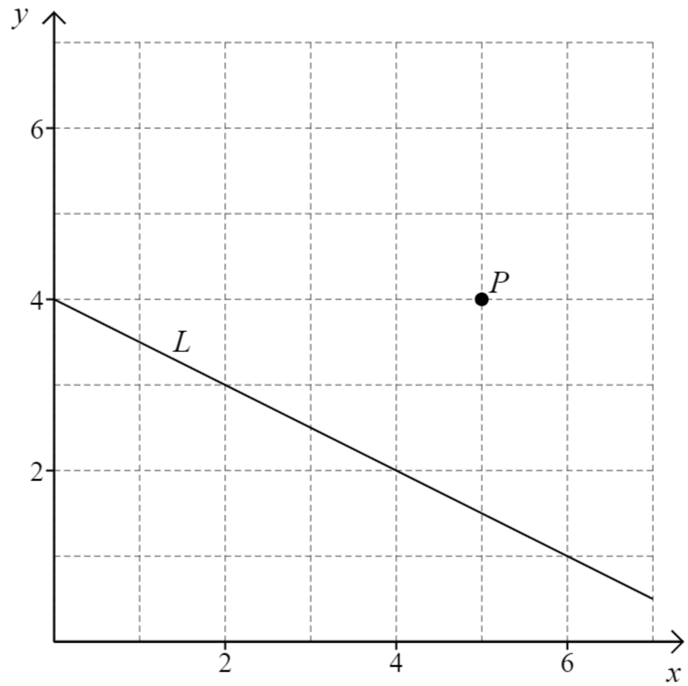
1. *Level 1 − 2 [Length: 4 minutes]*

The diagram below shows line L with equation $y = -\frac{x}{2} + 4$ and point P with coordinates $(5,4)$.



(a) Write down the coordinates of the closest point on line L to point P. [2]

(b) Hence calculate the exact distance from point P to line L. [2]

2. *Level 3 − 4* *[Length: 5 minutes]*

Consider line segment AB for points $A(-4,6)$ and $B(0,2)$.

(a) Find [2]

(i) the coordinates of the midpoint

(ii) the gradient of the line segment

(b) Hence find the equation of the perpendicular bisector of line segment AB. Write your answer in the form $y = mx + c$ where m and c are integers to be determined. [3]

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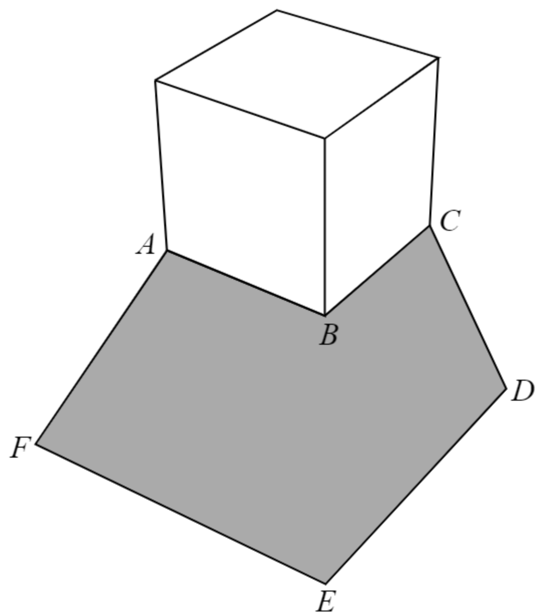
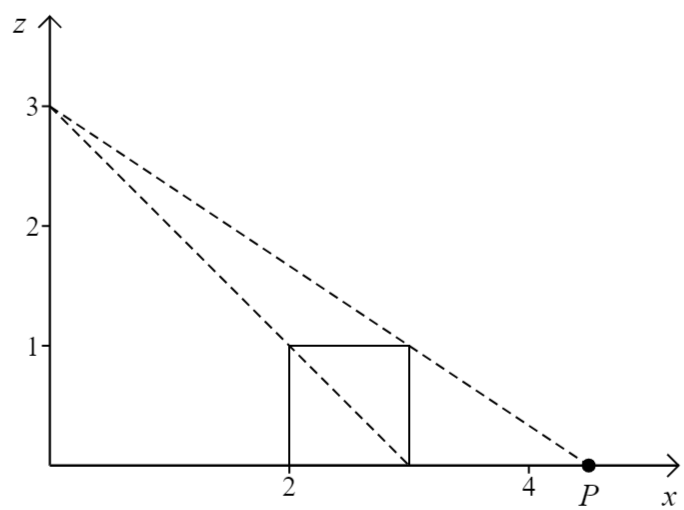
3. *Level 7 − 8 [Length: 13 minutes]*

A cube sitting on the xy-plane has vertices at points (2,1,0), (3,1,0), (3,2,0), (2,2,0), (2,1,1), (3,1,1), (3,2,1) and (2,2,1) where units of coordinates are metres. A light source at point (0,0,3) illuminates the cube causing it to cast a shadow onto the xy-plane.

This is shown in the diagram below. The six vertices of the shadow have been labelled.

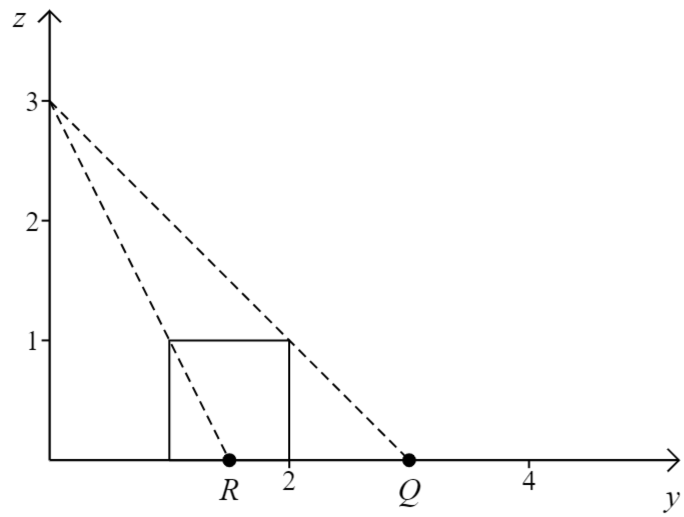


The diagram below shows the view perpendicular to the xz-plane.



(a) Find the x-coordinate of point P. [2]

The diagram below shows the view perpendicular to the yz-plane.



(b) Find the y-coordinate of [4]

(i) point Q

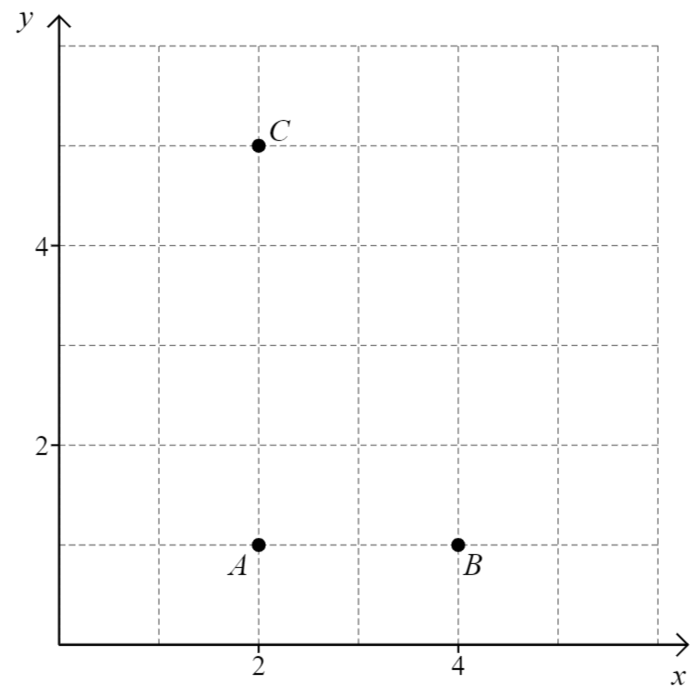
(ii) point R

(c) Hence write down the coordinates of points A, B, C, D, E and F. [4]

(d) Find the area of the shadow. [3]

4. *Level 5 − 6 [Length: 9 minutes]*

Three elementary schools are located at points $A(2,1)$, $B(4,1)$ and $C(2,5)$ where units of coordinates are kilometres. This is shown in the diagram below.



(a) Write down the equation of the perpendicular bisector of line segment [2]

(i) AB

(ii) AC

(b) Find the equation of the perpendicular bisector of line segment BC. Write your answer in the form $y = mx + c$ where m and c are constants to be determined. [4]

Children attend the school which they live the closest to.

(c) Add to the diagram above to create a Voronoi diagram. [2]

Children who live an equal distance to more than one school can choose between those schools.

(d) Write down the coordinates of a child's house who is able to choose from all three schools. [1]

5. *Level 3 − 4* *[Length: 5 minutes]*

Consider line segment AB for points $A(2,8)$ and $B(-2,10)$.

(a) Find [2]

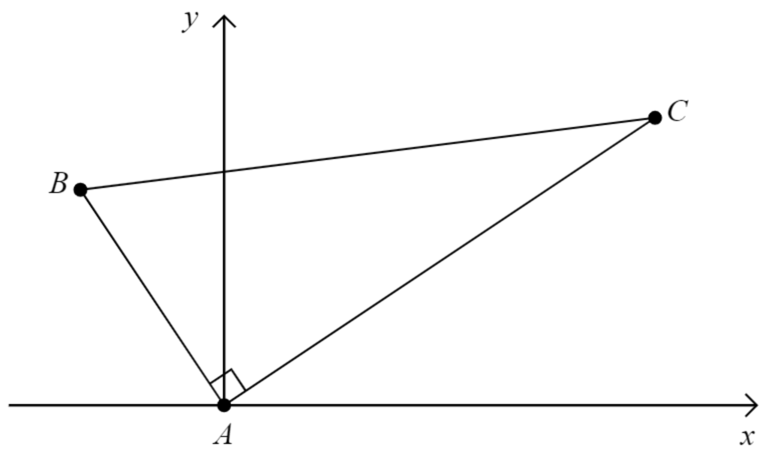
(i) the coordinates of the midpoint

(ii) the gradient of the line segment

(b) Hence find the equation of the perpendicular bisector of line segment AB. Write your answer in the form $y = mx + c$ where m and c are integers to be determined. [3]

6. *Level 7 − 8 [Length: 9 minutes]*

The diagram below shows a right-angled triangle with vertices at points $A(0,0)$, $B(x_1,y_1)$ and $C(x_2,y_2)$.



Let m_1 represent the gradient of line segment AB and d_1 represent its length.

(a) In terms of x_1 and y_1 write down [2]

(i) m_1

(ii) d_1

Let m_2 represent line segment AC and d_2 represent its length.

(b) In terms of x_2 and y_2 write down [2]

(i) m_2

(ii) d_2

(c) In terms of x_1, y_1, x_2 and y_2 write down the length of line segment BC. [1]

(d) Hence use the Pythagorean theorem to prove that $m_1 m_2 = -1$. [4]

1. (a) (4,2)

(b) $\sqrt{(5-4)^2+(4-2)^2}=\sqrt{5}$

2. (a)

(i) $\left(\dfrac{-4+0}{2},\dfrac{6+2}{2}\right)=(-2,4)$

(ii) $\dfrac{6-2}{-4-0}=\dfrac{4}{-4}=-1$

(b) The gradient of the bisector is equal to 1.

So we have

$$y-4=1(x+2)$$

Giving

$$y=x+6$$

3. (a) We have

$$\frac{P}{3} = \frac{P-3}{1}$$

Giving

$$P = 3P - 9$$

So

$$P = \frac{9}{2}$$

(b)

(i) We have

$$\frac{Q}{3} = \frac{Q-2}{1}$$

Giving

$$Q = 3Q - 6$$

So

$$Q = 3$$

(ii) We have

$$\frac{R}{3} = \frac{R-1}{1}$$

Giving

$$R = 3R - 3$$

So

$$R = \frac{3}{2}$$

(c) $A = (3,1,0)$

$B = (3,2,0)$

$C = (2,2,0)$

$D = (3,3,0)$

$E = \left(\frac{9}{2}, 3, 0\right)$

$F = \left(\frac{9}{2}, \frac{3}{2}, 0\right)$

(d) Divide the shadow into two trapezoids.

The area is then

$$\frac{1+\frac{3}{2}}{2} \times \frac{3}{2} + \frac{1+\frac{3}{2}}{2} \times 1 = \frac{25}{8} = 3.125$$

4. (a)

(i) $x = 3$

(ii) $y = 3$

(b) The coordinates of the midpoint are (3,3).

The gradient is $\frac{1}{2}$.

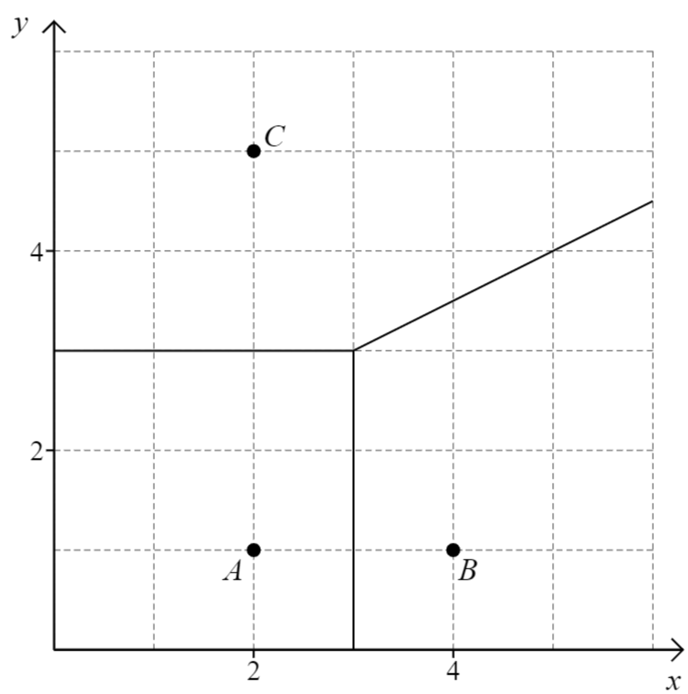
So the equation is $y - 3 = \frac{1}{2}(x - 3)$

Giving

$$y = \frac{x}{2} + \frac{3}{2}$$

(c) The lines are added correctly.

They meet at point (3,3).



(d) (3,3)

5. (a)

(i) $\left(\frac{2-2}{2}, \frac{8+10}{2}\right) = (0,9)$

(ii) $\frac{10-8}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$

(b) The gradient of the bisector is equal to 2.

So we have

$$y - 9 = 2(x - 0)$$

Giving

$$y = 2x + 9$$

6. (a)

(i) $\frac{y_1}{x_1}$

(ii) $\sqrt{x_1{}^2 + y_1{}^2}$

(b)

(i) $\frac{y_2}{x_2}$

(ii) $\sqrt{x_2{}^2 + y_2{}^2}$

(c) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(d) We have

$$x_1{}^2 + y_1{}^2 + x_2{}^2 + y_2{}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Expand

$$x_1{}^2 + y_1{}^2 + x_2{}^2 + y_2{}^2 = x_1{}^2 + y_1{}^2 - 2x_1x_2 + x_2{}^2 + y_2{}^2 - 2y_1y_2$$

Simplify

$$x_1x_2 = -y_1y_2$$

Giving

$$\frac{y_1}{x_1} \cdot \frac{y_2}{x_2} = -1$$